VNU Journal of Science, Mathematics - Physics 23 (2007) 168-177

Filtering for stochastic volatility from point process observation

Tidarut Plienpanich[1], Tran Hung Thao[2,*]

[1]*School of Mathematics, Suranaree University of Technology, 111 University Avenue, Muang District, Nakhon Ratchasima, 30000, Thailand*
[2]*Institute of Mathematics, 18 Hoang Quoc Viet, Cau Giay, Hanoi, Vietnam*

Received 15 November 2006; received in revised form 12 September 2007

Abstract. In this note we consider the filtering problem for financial volatility that is an Ornstein-Ulhenbeck process from point process observation. This problem is investigated for a Markov-Feller process of which the Ornstein-Ulhenbeck process is a particular case.
Keywords: and phrases: filtering, volatility, point process. AMSC 2000: 60H10; 93E05.

Introduction and notations

Stochastic volatility is one of main objective to study of financial mathematics. It reflects qualitively random effects on change of financial derivatives, interest rate and other financial product prices.

Many results have been received recently for volatility estimation by filtering approach. Rüdiger Frey and W. J. Runggaldier [1] studied for the case of high frequency data. Frederi G. Viens [2] considered the problem of portfolio optimization under partially observed stochastic volatility. Wolfgang J. Runggaldier [3] used filtering methods to specify coefficients of financial market models.

A filtering approach was introduced by J. Cvitanic, R. Liptser and B. Rozovskii [4] to tracking volatility from prices observed at random times. A filtering problem for Ornstein-Ulhenbeck signal from discrete noises was investigated by Y.Zeng and L.C.Scott [5] to applied to the micro-movement of stock prices. Also a practical method of filtering for stochastic volatility models was given by J. R. Stroud, N. G. Polson and P. Müller [6].

These authors introduced also a sequential parameter estimation in stochastic volatility models with jumps [7]. And other contributions were given recently by A. Bhatt, B. Rajput and Jie Xiong, R. Elliott, R. Mikulecivius and B, Rozovskii.

Filtered multi-factor models are studied by E. Platen and W. J. Runggaldier [8] by a so-called benchmark approach to filtering.

1. Filtering from point process observation

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space on which all processes are defined and adapted to a filtration $(\mathcal{F}_t,\ \ t \geq 0)$ that is supposed to satisfy " usual conditions".

* Corresponding author. E-mail: ththao@math.ac.vn

For the sake of simplicity, all stochastic processes considered here are supposed to be 1-dimensional real processes.

We consider a filtering problem where the signal processes is a semimartingale

$$X_t = X_0 + \int_0^t H_s ds + Z_t, \tag{1}$$

where Z_t is a square integrable $\mathcal{F}_t$- martingale, H_t is bounded $\mathcal{F}_t$-progressive process and $E[\sup_{s\leq t}|X_s|] < \infty$ for every $t \geq 0$, X_0 is a random variable such that $E|X_0|^2 < \infty$; the observation is given by a point process $\mathcal{F}_t$- semimartingale of the form

$$Y_t = \int_0^t h_s ds + M_t, \tag{2}$$

where M_t is a square integrable $\mathcal{F}_t$-martingale with mean 0, $M_0 = 0$ such that the future σ- field $\sigma(M_u - M_t; u \geq t)$ is independent of the past one $\sigma(Y_u, h_u; u \leq t)$, $h_t = h(X_t)$ is a positive bounded $\mathcal{F}_t$- progressive process such that $E\int_0^t h_s^2 ds < \infty$ for every t.

Denote by $\mathcal{F}_t^Y$ the σ-algebra generated by all random variables $Y_s, s \leq t$. Thus $\mathcal{F}_t^Y$ records all information about the observation up to the time t.

Suppose that the process $u_s = \frac{d}{ds} < Z, M >_s$ is $\mathcal{F}_s$- predictable ($s \leq t$) where $<,>$ stands for the quadratic variation of Z_t and M_t. Denote also by $\hat{u}_s$ the $\mathcal{F}_t^Y$- predictable projection of u_s. By assumptions imposed on Z and M we see that $< Z, M >= 0$, so $u_s = 0$.

The filter of (X_t) based on information given by (Y_t) is defined as the conditional expectation

$$\pi(X_t) := E(X_t|\mathcal{F}_t^Y), \tag{3}$$

or more general

$$\pi_t(f) := E[f(X_t)|\mathcal{F}_t^Y], \tag{4}$$

where f is a bounded continuous function $f \in C_b(\mathbb{R})$.

Denote by $\pi(h_t)$ the filtering process corresponding to the process h_t in (2).

Let m_t be a process defined by

$$m_t = Y_t - \int_0^t \pi(h_s)ds. \tag{5}$$

The process m_t is called the innovation from the observation process Y_t.

Lemma 1.1. *m_t is a point process $\mathcal{F}_t^Y$-martingale and for any t, the future σ-field $\sigma(m_t - m_s\ ;\ t \geq s)$ is independent of $\mathcal{F}_s^Y$.*

Proof. We have by definitions (2) and (5):

$$\begin{aligned} m_t - m_s &= Y_t - Y_s - \int_s^t \pi(h_u)du \\ &= M_t - M_s + \int_s^t [h_u - \pi(h_u)]du. \end{aligned} \tag{6}$$

It follows from assumption of M_t that

$$E[(M_t - M_s)|\mathcal{F}_s^Y] = 0. \tag{7}$$

On the other hand, since for $u \geq s$

$$E(h_u|\mathcal{F}_s^Y) = E[E(h_u|\mathcal{F}_u^Y)|\mathcal{F}_s^Y] = E[\pi(h_u)|\mathcal{F}_s^Y],$$

or

$$E[\int_s^t [h_u - \pi(h_u)]du|\mathcal{F}_s^Y] = 0, \tag{8}$$

and then

$$E[m_t - m_s|\mathcal{F}_s^Y] = 0\ ,\ t \geq s. \tag{9}$$

Now for any s, t such that $0 \leq s \leq t$ we consider two families $\mathcal{C}_t$ and $\mathcal{D}_t$ of sets of random variables defined as follows:

$$\mathcal{C}_{s,t} = \{\text{sets } C_a\ , s \leq a \leq t\} \text{ where } C_a = \{m_t - m_\alpha\,; a \leq \alpha \leq t\}$$

$$\mathcal{D}_s = \{\text{sets } D_b, 0 \leq b \leq t\} \text{ where } D_b = \{Y_\beta\,; b \leq \beta \leq s\}.$$

It is easy to check that $\mathcal{C}_{s,t}$ and $\mathcal{D}_s$ are π-systems, i.e. they are closed under finite intersections. Also they are independent each of other by (9). It follows that (refer to [9]) the σ-algebra $\sigma(\mathcal{C}_{s,t}) = \sigma(m_t - m_s, s \leq t)$ generated by $\mathcal{C}_{s,t}$ is independent of σ-algebra $\sigma(\mathcal{D}_s) = \mathcal{F}_s^Y$ generated by $\mathcal{D}_s$. The second assertion of Lemma 1.1 as thus established.

We state here an important result by P. Bremaud on an integral representation for $\mathcal{F}_t^Y$-martingale:

Lemma 1.2. *Let R_t be a $\mathcal{F}_t^Y$-martingale. Then there exists a $\mathcal{F}_t^Y$-predictable process K_t such that for all $t \geq 0$,*

$$\int_0^t K_s \pi(h_s)ds < \infty \ P.a.s, \tag{10}$$

and such that R_t has the following representation:

$$R_t = R_0 + \int_0^t K_s dm_s. \tag{11}$$

Remark. Since the innovation process m_t is a $\mathcal{F}_t^Y$- martingale so it can represented by

$$m_t = m_0 + \int_0^t K_s dm_s, \tag{12}$$

where K_t is some $\mathcal{F}_t^Y$- predictable process satisfying (10). It is known from [10] that K_t is of the form

$$K_t = \pi(h_t)^{-1}[\pi(X_{t}\text{-}h_t) - \pi(X_{t}-)\pi(h_t) + \hat{u}_t],$$

and since $\hat{u}_t = 0$ we have

Theorem 1.1. *The filtering equation for the filtering problem (1)- (2) is given by:*

$$\pi(X_t) = \pi(X_0) + \int_0^t \pi(H_s)ds + \int_0^t \pi^{-1}(h_s)[\pi(X_{s}\text{-}h_s) - \pi(X_{s}-)\pi(h_s)]dm_s. \tag{13}$$

provided $\pi(h_t) \neq 0$ a.s.

Remark. If the observation is given by a standard Poisson process Y_t then the filtering equation takes the following form

$$\pi(X_t) = \pi(X_0) + \int_0^t \pi(H_s)ds + \int_0^t \pi^{-1}(h_s)X_{s^-}[\pi(h_s) - 1]dm_s, \tag{14}$$

where $m_t = Y_t - t$.

Quasi-filtering. There is some inconvenience in application of (13) because the appearance of the factor$[\pi(h_s)]^{-1}$. To avoid this difficulty we introduce the unnormalized conditional filtering or quasi-filtering in other term.

As we know in the method of reference probability, the probability P actually governing the statistics of the observation Y_t is obtained from a probability Q by an absolutely continuous change $Q \to P$. We assume that Q is the reference probability such that Y is a $(Q, \mathcal{F}_t)$- Poisson process of intensity 1, where $\mathcal{F}_t = \mathcal{F}_t^Y \vee \mathcal{F}_\infty^X$.

Denoting for every $t \geq 0$ by P_t and Q_t the restrictions of P and Q respectively to $(\Omega, \mathcal{F}_t)$ we have $P_t << Q_t$. It is known that the corresponding Radon-Nykodym derivative is the unique solution of a Doleans-Dade equation:

$$L_t = 1 + \int_0^t L_{s-}(h_s - 1)dM_s, \tag{15}$$

where h_t and M_t are given in (2).

The explicit solution of (15) is

$$L_t = \frac{dP_t}{dQ_t} = \Pi_{0\leq s\leq t} h_s \Delta Y_s \exp \int_0^t (1 - h_s)ds. \tag{16}$$

Let Z_t be a real valued and bounded process adapted to $\mathcal{F}_t$, then for every history $\mathcal{G}_t$ such that $\mathcal{G}_t \subseteq \mathcal{F}_t$, $t \geq 0$ we have a Bayes formula

$$E_P(Z_t|\mathcal{G}_t) = \frac{E_Q(Z_t L_t|\mathcal{G}_t)}{E_Q(L_t|\mathcal{G}_t)}, \tag{17}$$

where $E_P(.|\mathcal{G}_t)$ and $E_Q(.|\mathcal{G}_t)$ are conditional expectations under probabilities P and Q respectively.
Definition. The process $\sigma(X_t)$ defined by

$$\sigma(X_t) = E_Q(L_t X_t|\mathcal{F}_t) \tag{18}$$

is call the optimal quasi-filter (or quasi-filter) of X_t based on data $\mathcal{F}_t$. It is in fact an unnormalized filter of X_t.
Remarks.

(i) If under the probability Q, Y_t is a standard Poisson process (i.e of intensity 1) and the process $\mu_t \equiv Y_t - t$ is then a $(\mathcal{F}_t,\ Q)$-martingale.

(ii) We have by consequence of the definition

$$\pi(X_t) = \frac{\sigma(X_t)}{\sigma(1_t)}, \tag{19}$$

where 1 stands for function identified to for every t: $1(t) \equiv 1$.

Replacing $\pi(.)$ by its expression given by (19) we can rewrite the filtering equation (14) as an equation for quasi-filtering $\sigma(.)$:

Theorem 1.2. *The assumptions are those prevailing in Theorem 1.1. Moreover, assume that Z_t and M_t have no common jumps. Then the quasi-filter $\sigma(X_t)$ satisfies the following equation*

$$\sigma(X_t) = \sigma(X_0) + \int_0^t \sigma(H_s)ds + \int_0^t [\sigma(X_{s}\text{-}h_s) - \sigma(X_s-)]dn_s, \tag{20}$$

where

$$n_t = Y_t - t\,. \tag{21}$$

Proof. Suppose we have (13) already:

$$\pi(X_t) = \pi(X_0) + \textstyle\int_0^t H(X_s)ds + \int_0^t \pi^{-1}(h_s)\gamma_s dm_s \tag{13'}$$

where $\gamma_s = \pi(X_{s}\text{-}\, h_s) - \pi(X_s-)\pi(h_s)$ and $m_s = Y_s - \int_0^t \pi(h_s)ds$.

By definition $\sigma(X_t) = \pi(L_t)\pi(X_t)$. Applying a formula of integration by part we get

$$\begin{aligned}\pi(L_t)\pi(X_t) = \pi(X_0) + \int_0^t \pi(X_s)\pi(H_s)ds + \int_0^t \pi(L_s-)\gamma_s dm_s \\ + \int_0^t \pi(X_s-)\pi(L_s-)[\pi(h_s) - 1]dn_s + [\pi(L), \pi(X)]_t\end{aligned} \tag{22}$$

where $n_t = Y_t - t$ and $[.,.]$ stands for the quadratic variation.

Because $\pi(X_0) = \sigma(X_0)$ and there are at most countably many points where $\pi(L_t-) \neq \pi(L_t)$ so

$$\int_0^t \pi(L_s-)\pi(H_s)ds = \int_0^t \pi(L_s)\pi(H_s)ds = \int_0^t \sigma(H_s)ds.$$

On the other hand we have

$$[\pi(L), \pi(X)]_t = \sum_{0 \le s \le t} \Delta\pi(L_s)\Delta\pi(X_s) = \int_0^t \gamma_s \pi(h_s-)[\pi(h_s) - 1]dY_s. \tag{23}$$

Then

$$\begin{aligned}\pi(L_t)\pi(X_t) = \sigma(X_t) = \sigma(X_0) + \int_0^t \sigma(H_s)ds + \\ + \int_0^t \pi(L_s-)\big[\pi(X_{s}\text{-}h_s) - \pi(X_s)\pi(h_s)\big]dn_s \\ + \int_0^t \pi(L_s-)\pi(X_s-)\big[\pi(h_s) - 1\big]dn_s\end{aligned}$$

$$= \sigma(X_0) + \int_0^t \sigma(H_s)ds + \int_0^t \big[\sigma(X_{s}\text{-}h_s) - \sigma(X_s-)\big]dn_s\,. \tag{24}$$

The proof of Theorem 1.2 is thus completed.

2. Filtering for a Fellerian system

Suppose that X_t is a Markov process taking values in a compact separable Hausdorff space S and that the semigroup $(P_t\ ,\ t \geq 0)$ associated with the transition probability $P_t(x, E)$ is a Feller semigroup, that is

$$P_t f(x) = \int_0^t P_t(x, dy) f(y), \tag{25}$$

maps $C(S)$ into itself for all $t \geq 0$ satisfies

$$\lim_{t \downarrow 0} P_t f(x) = f(x), \tag{26}$$

uniformly in S for all $f \in C(S)$, where $C(S)$ is the space of all real continuous function over S. Assume that the observation Y_t is a Poisson process of intensity $h_t = h(X_t) \in C(S)$.

As before the filter π_t is defined as:

$$\pi_t(f) = \pi(f(X_t)) := E[f(X_t)|\mathcal{F}_t^Y]. \tag{27}$$

Also we have

$$\sigma_t(f) := \sigma(f(X_t)) = E_Q[L_t f(X_t)|\mathcal{F}_t^Y], \tag{28}$$

where the probability Q and the likelihood ratio are defined as in subsection 1.2.

Denote by m_t the innovation process of Y_t:

$$m_t := Y_t - \int_0^t \pi_s(h)ds = Y_t - \int_0^t \frac{\sigma_s(h)}{\sigma_s(1)}ds. \tag{29}$$

The following results are given in [8]:

Theorem 2.1 *[Filtering equation for Feller process with point process observation] If A is infinitesimal generator of the semigroup P_t of the signal process, then the optimal filter $\pi_t(f) = \pi(f(X_t))$ satisfies the two following equations provided $\pi_s(h) \neq 0$ a.s.*

a)

$$\begin{aligned}\pi_t(f) &= \pi_0(f) + \int_0^t \pi_s(Af)ds + \\ &\quad + \int_0^t \pi_s^{-1}(h)[\pi_{s^-}(fh) - \pi_{s^-}(f)\pi_s(h)]dm_s\ ,\ f \in C_b(S),\end{aligned} \tag{30}$$

b)

$$\begin{aligned}\pi_t(f) &= \pi_0(P_t f) + \int_0^t \pi_s^{-1}(h)[\pi_{s^-}(hP_{t-s}f) \\ &\quad -\pi_{s^-}(P_{t-s}f)\pi_s(h)]dm_s\ , f \in C_b(S).\end{aligned} \tag{31}$$

Theorem 2.2 [Quasi-filtering equation for Feller process with point process observation]. *The quasi-filter σ_t satisfies the two following equations:*

a)

$$\sigma_t(f) = \sigma_0(f) + \int_0^t \sigma_s(Af)ds + \int_0^t [\sigma_{s-}(hf) - \sigma_{s-}(f)]dm_s\ ,\ f \in C_b(S), \tag{32}$$

b)

$$\sigma_t(f) = \sigma_0(P_t f) + \int_0^t [\sigma_{s-}(hP_{t-s}f) - \sigma_{s-}(P_{t-s}f)]dm_s \; f \in C_b(S). \tag{33}$$

3. Ornstein- Ulhenbeck process and financial filtering

We recall in this Section some facts on Ornstein- Ulhenbeck and show how to use it to our filtering problems. This process is of importance in studies in finance. It has various 'good properties' to describe many elements in financial models as that of interest rate (Vacisek, Ho-Lee, Hull-White, etc.) or stochastic volatility of asset pricing.

Let $X = (X_t, t \geq 0)$ be a stochastic process with initial value X_0 of standard normal distributed: $X_0 \in \mathcal{N}(0,1)$.

3.1. Definition. If (X_t) is a Gaussian process with

a) mean $EX_t = 0$, $\forall t \geq 0$

b) Covariance function

$$R(s,t) = E(X_s X_t) = \gamma \exp(-\alpha|t-s|) \;, s,t \geq 0; \quad \alpha, \gamma \in \mathbb{R}^+, \tag{34}$$

then X_t is called an Ornstein-Ulhenbeck.

It follows from this definition that (X_t) is a stationary process in wide-sense. It is also a stationary process in strict sense since its density of the transition probability is given by

$$p(s,x;t,y) = \frac{1}{\sqrt{\gamma\pi(1-e^{-2\alpha(t-s)})}} \exp\left\{ -\frac{(y - xe^{-2\alpha(t-s)})^2}{\gamma(1-2e^{-2\alpha(t-s)})} \right\}, \tag{35}$$

that depends only on $(t-s)$, where γ is some positive constant.

3.2. Stochastic Langevin equation. An Ornstein-Ulhenbeck (X_t) can be defined also as the unique solution of the form

$$dX_t = -\alpha X_t dt + \gamma dW_t \;, X_0 \sim \mathcal{N}(0,1), \tag{36}$$

where $\alpha > 0$ and γ are constants.

The explicit form of this solution is

$$X_t = X_0 e^{-\alpha t} + \gamma \int_0^t e^{-\alpha(t-s)} dW_s,$$

and its expectation, variance and covariance are given by

$$EX_t = e^{-\alpha t} \;,$$

$$V_t := Var(X_t) = \frac{\gamma^2}{2\alpha} \;,$$

$$R(s,t) = \frac{\gamma^2}{2\alpha} e^{-\alpha|t-s|},$$

where $\frac{\gamma^2}{2\alpha}$ is denoted by β in (34)

3.3. Ornstein - Ulhenbeck process as a Feller process. Consider a standard Gaussian measure on R

$$\mu(dx) = \frac{1}{\sqrt{2\pi}} \exp\left(- \frac{x^2}{2} \right) dx.$$

It is known that an Orntein - Ulhenbeck process (X_t) is a Markov process and its semigroup is defined by a family $(P_t \ , t \geq 0)$ of operations on bounded Borelian functions f:

$$(P_t f)(x) = \int_R f(e^{-\alpha t} x + \frac{\gamma^2}{2\alpha} \sqrt{1 - e^{-2\alpha t}} y) \mu(dy). \tag{37}$$

It is obvious that

$$\lim_{t \downarrow 0} (P_t f)(x) = f(x), \tag{38}$$

then X_t is really a Feller process and the family $(P_t, t \geq 0)$ is called an Ornstein- Ulhenbeck semigroup.

3.4. Filtering for Ornstein-Ulhenbeck process from point process observation. We will apply results of Section II to the following filtering problem:

- Signal process: An Ornstein-Ulhenbeck process X_t that is solution of the equation (36).
- Observation process: A point process N_t of intensity $\lambda_t > 0$.

So the signal and observation processes (X_t, N_t) can be expressed in the form

$$dX_t = -\alpha X_t dt + \gamma dW_t \ , X_0 \sim \mathcal{N}(0, 1), \tag{39}$$

$$dN_t = \lambda_t dt + M_t, \tag{40}$$

where $\alpha, \gamma > 0$, λ_t is a $\mathcal{F}_t$-adapted process, M_t is a point process martingale independent of W_t.

Denote by $\mathcal{F}_t^N$ the σ-algebra of observation that is generated by $(N_s, s \leq t)$

The filter of (X_t) based on data given by $(\mathcal{F}_t^N)$ is denoted now by $\hat{X}_t$:

$$\hat{X}_t = \pi_t(X) = E(X_t | \mathcal{F}_t^Y)$$

and also $\pi_t(f) = f(\hat{X}_t) = E(f(X_t)|\mathcal{F}_t^Y) \ , f \in C_b(R)$.

The innovation process m_t is given by

$$m_t = Y_t - \int_0^t \hat{\lambda}_t ds, \tag{41}$$

and $dm_t = dY_t - \hat{\lambda}_t dt$.

Since the semigroup $(P_t \ , t \geq 0)$ for X_t is defmed by (37), the infinitesimal operator A_t is given by

$$A_t f = \lim_{t \to 0} \frac{1}{t}(P_t f - f) = -\alpha x f'(x) + \frac{1}{2\alpha} \gamma^2 f''(x). \tag{42}$$

On the other hand, $P_t f$ can be expressed under the form:

$$(P_t f)(x) = E[f(e^{-\alpha t} x + \frac{\gamma^2}{2\alpha} \sqrt{1 - e^{-2\alpha t}} Y)], \tag{43}$$

where Y is a standard gaussian variable, $Y \sim \mathcal{N}(0, 1)$.

Then from Theorem 2.1 we can get:

Theorem 3.1. *a)*

$$\pi_t(f) = \pi_0(f) + \int_0^t \pi_s[-\alpha X f'(X) + \frac{\gamma^2}{2\alpha} f''(X)]ds + \int_0^t \pi_s^{-1}(\lambda)[\pi_{s^-}(\lambda f) - \pi_{s-}(f)\pi_s(\lambda)](dY_s - \pi_s(\lambda)ds), \quad (44)$$

b)

$$\pi_t(f) = \pi_0(P_t f) + \int_0^t \pi_s^{-1}(\lambda)[\pi_{s^-}(\lambda P_{t-s} f) - \pi_{s-}(P_{t-s} f)\pi_s(\lambda)][dY_s - \pi_s(\lambda)ds], \quad (45)$$

where P_t is given by (43).

Theorem 3.2. *The quasi-filter $\sigma_t(f)$ for the filtering (39)- (40) is given by one of two following equations:*

a)

$$\sigma_t(f) = \sigma_0(f) + \int_0^t \sigma_s[-\alpha X f'(X) + \frac{\gamma^2}{2\alpha} f''(X)]ds + \int_0^t [\sigma_{s-}(\lambda f) - \sigma_{s-}(f)][dY_s - \pi_s(\lambda)ds], \quad (46)$$

b) $\sigma_t(f) = \sigma_0(P_t f) + \int_0^t [\sigma_{s-}(\lambda P_{t-s} f) - \sigma_{s-}(P_{t-s} f)][dY_s - \pi_s(\lambda)ds].$

The fisrt author was supported by the Royal Golden Jubilee Ph.D Program of Thailand (TRF).

Remarks.

(i) The above results can be applied also to term structure models for interest rates, where the rate is expressed as an Orstein-Ulhenbeck process and the observation is given by a point process of form

$$N_t = \int_0^t h(S_s)ds + M_t\,, \quad 0 \le t \le T,$$

where S_t is the a process observed stock prices the models for Vacisek, Ho-Lee, Hull-White ... can be included in this context.

(ii) The assumption that the volatility of asset pricing is of form of an Ornstein-Ulhenbeck process is quite frequently met in various financial models. So above results can give another approach to estimate this volatility.

Acknowledgements. This paper is based on the talk given at the Conference on Mathematics, Mechanics, and Informatics, Hanoi, 7/10/2006, on the occasion of 50th Anniversary of Department of Mathematics, Mechanics and Informatics, Vietnam National University, Hanoi.

References

[1] R. Frey, W.J. Runggaldier, A Nonlinear Filtering Approach to Volatility Estimation with a View Towards High Frequency Data, *International Journal of Theoretical and Applied Finance* 4 (2001) 199.

[2] F.G. Viens, *Portfolio Optimization Under Partially Observed Stochastic Volatility*, Preprint, Dept of Statistics and Dept. of Math., Perdue University, West Lafayelte, US (2000).

[3] W.J Runggaldier, Estimation via Stochastic Filtering in Financial Market Models,*Mathematics of Finance (G. Yin and Q. Zhang Eds.), Contemporary Mathematics* Vol.351, American Mathematical Society, providence R.I., (2004) 309.

[4] J. Cvitanic, R. Liptser, B. Rozovskii B, A Filtering Approach to Tracking Volatility from Prices Observed at Random Times, *The Annals of Applied Probability*, vol. 16, no. 3 (2006).

[5] Y. Zeng, L.C. Scott, *Bayes Estimation via Filtering Equation for O-U Process with Discrete Noises: Application to the Micro-Movements of Stock Prices, Stochastic Theory and Control* (Bozenna Pasik-Duncan Ed.), Lecture Notes in Control and Information Sciences, Springer, (2002) 533.

[6] J.R. Stroud, N.G. Polson N.G, P. Müller, Practical Filtering for Stochastic Volatility Models, *State Space and Unobserved Components Models* (Harvey, Koopmans and Shephard, Eds.) (2004) 236.

[7] M. Johannes, N.G. Polson, J. Stroud, *Nonlinear Filtering of Stochastic Differential Equations with Jumps*, Working paper, Univ. of Columbia NY, Univ. of Chicago and Univ. of Pennsylvania, Philadelphia (2002).

[8] E. Platen, W.J. Runggaldier, A Benchmark Approach to Filtering in Finance, *Financial Engineering and Japanese markets*, vol. 11, no. 1 (2005) 79.

[9] O. Kallenberg, *Foundation of Modern Probability*, Springer, 2002.

[10] T. H. Thao, Optimal State Estimation of a Markov Process From Point Process Observations, *Annales Scientifiques de l' Université Blaise Pascal, Clermont-Ferrand II, Fasc.* 9 (1991) 1.